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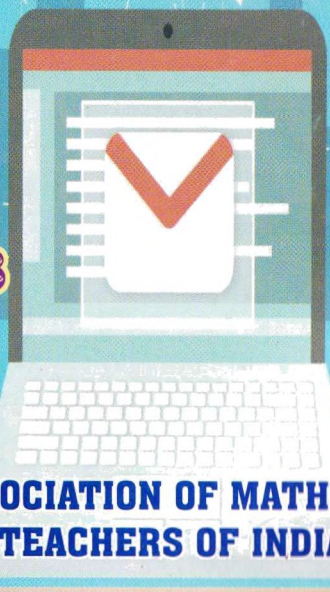
AMTI

THE MATHEMATICS TEACHER



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THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

The Association of Mathematics Teachers of India (AMTI) was started in 1965 for the promotion of efforts to improve Mathematics education at all levels. A major aim of the Association is to assist practising teachers of Mathematics in schools in improving their expertise and professional skills. Another important aim is to spot out and foster Mathematical talents in the young. The Association also seeks to disseminate new trends in Mathematics education among parents and public. Other activities of the Association include consultancy services to schools in equipping the Mathematics section of their libraries, in organising children's Mathematics clubs and fairs, in setting up teacher centres in schools, in conducting Mathematics laboratory programmes, in holding practical tests in Mathematics in assisting children in participating investigational projects.

The Association holds " National Mathematical Talent Search Competition " annually and organizes Orientation Courses, Seminars and Workshops for teachers and courses for talented students. A national conference is held annually in different parts of the country for teachers to meet and deliberate on important issues of Mathematics education. Innovative teacher award has been instituted to give public recognition to enterprising and pioneering teachers of Mathematics for which entries from teachers are invited.

An award for contributions to the Mathematics Teacher relating to History of Mathematics in the context of mathematics education had been instituted by Prof.R.C.Gupta.

"The Mathematics Teacher (India)" (MT) is the official quarterly journal of the Association and is issued twice a year. It has been approved for use in schools and colleges of education by the Government departments of education in many States. Besides MT the Association also brings out Junior Mathematician (JM), three issues in a year, especially for school students in English and Tamil.

The membership of the Association is open to all those interested in Mathematics and Mathematics Education. The membership fee inclusive of subscription for "**The Mathematics Teacher (India)**" and effective from April 1993 is as follows:

Subscription for India*

Category	Individual	Institutional
Life	Rs. 500	Rs. 1000
Annual (Ordinary)	Rs. 50	—
Junior Mathematician - Life	Rs. 250	Rs. 250
Junior Mathematician - Annual	Rs. 25	Rs. 25

The Journal "The Mathematics Teacher" will be supplied free to all members.
Fifty or more subscriptions to Junior Mathematician will entail 20% discount.

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(Inclusive of postage)
i.e instead of rupees read US dollars

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Dr. S.Muralidharan
Editor

EDITORIAL

This issue contains the texts of some of the talks delivered at the 52nd Annual Conference of AMTI conducted between December 27th and 29th December 2017 at Jain Heritage School, Bangalore.

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Get Your Students Fall in Love [1]

S Muralidharan
Tata Consultancy Services
Chennai

It is the first hour in the school. The teacher enters the class and the students greet her with "Good Morning" and settle down. The teacher starts: "Children, we are going to learn a new topic today. It is Cricket. This is a popular game and two teams with eleven players each play against one another. The teams use a ball and a bat. The game requires several skills and we will learn them one by one. Today, we will learn how to do a "Up Throw". (Showing a tennis ball), "Children, this is a ball and we do an Up Throw like this (demonstrates throwing the ball upwards). Now all of you practice 5 Up Throws". Seeing Rahul sitting quiet, she shouts, "Rahul, why are you not doing Up Throws?" ... After this "learning experience", is it a wonder nobody gets interested in Cricket?

Am I exaggerating? Perhaps. But how do we teach Mathematics today? Invariably, we teach the *procedure*. In essence, we teach to follow directions. Undoubtedly, following directions is an important skill – specially, after you get married, you need this skill every day to live peacefully. We introduce a concept or method, solve some example problems and ask students practice along the same lines. Most of the problems are just number variations of the examples and can be done using "plug and play" in the method shown by the

[1] Prof. A Narasinga Rao Memorial Lecture at 52nd Annual Conference Association of Mathematics Teachers of India (AMTI)

teacher. Students wonder why should they learn all this stuff, are these used anywhere at all and when they are unable to get the hang of it, doubt if they are alone in not understanding. Quickly, they come to the conclusion, "Math is not for me".

In any movie, there are three main aspects – Script, Screen play and the Director. Script contains the story line and screen play is an expansion of the story scene by scene. The Director visualizes the scene in his mind and expands on it to give the viewers a stunning experience. One of the key ingredient of success of a movie is to get you engrossed in the movie from scene one. How the main character is introduced is very important and all directors realize this. When you see Bahubali II, the hero gets introduced in a spectacular visual. Thus the director gets the viewers riveted to the screen and

scene after scene, your amazement increases. When the screen shows "Intermission", you wonder how quickly those 65 minutes passed by. That is the success of the screen play and the director.

Mathematics class can be made as engrossing as a movie. The script is the text book. But the way the teacher (who is also the

DIFFERENTIAL EQUATIONS

❖ *He who seeks for methods without having a definite problem in mind seeks for the most part in vain.* – *D. HILBERT* ❖

9.1 Introduction

In Class XI and in Chapter 5 of the present book, we discussed how to differentiate a given function f with respect to an independent variable, i.e., how to find $f'(x)$ for a given function f at each x in its domain of definition. Further, in the chapter on Integral Calculus, we discussed how to find a function f whose derivative is the function g, which may also be formulated as follows:

For a given function g, find a function f such that

$$\frac{dy}{dx} = g(x), \text{ where } y = f(x) \quad \ldots (1)$$

An equation of the form (1) is known as a ***differential equation***. A formal definition will be given later.

Henri Poincare
(1854-1912)

These equations arise in a variety of applications, may it be in Physics, Chemistry, Biology, Anthropology, Geology, Economics etc. Hence, an indepth study of differential equations has assumed prime importance in all modern scientific investigations.

In this chapter, we will study some basic concepts related to differential equation, general and particular solutions of a differential equation, formation of differential equations, some methods to solve a first order - first degree differential equation and some applications of differential equations in different areas.

9.2 Basic Concepts

We are already familiar with the equations of the type:

$$x^2 - 3x + 3 = 0 \quad \ldots (1)$$
$$\sin x + \cos x = 0 \quad \ldots (2)$$
$$x + y = 7 \quad \ldots (3)$$

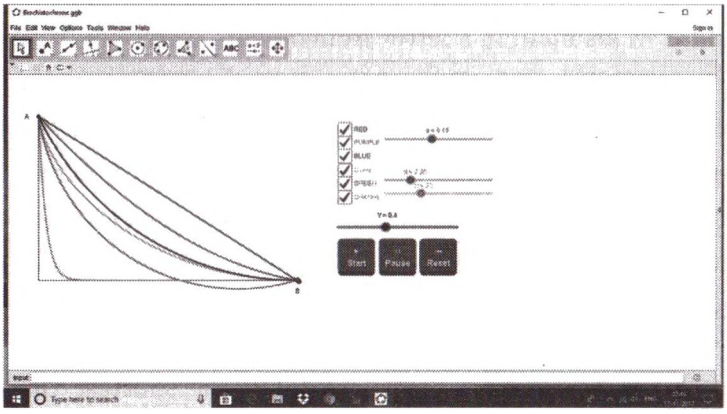
Director and actor) writes the screen play makes a significant difference. Unfortunately, more often than not, the script lacks imagination. For example, look at the "script" for Differential Equations in the NCERT text book, shown below. There is a picture of Henri Poincare, but there is no mention of why it appears there. In the historical note at the end of the chapter, we are told that it was Henri Poincare who strongly advocated the use of the word "solution" for what used to be called

"Integral of a differential equation". Also, we are told "These equations arise in a variety of applications, may it be in Physics, Chemistry, Biology, Anthropology, Geology, Economics etc. Hence, an indepth study of differential equations has assumed prime importance in all modern scientific investigations". Not one specific application is given! The student is supposed to take all this on faith. Ironically, the chapter starts with the quote from Hilbert: "He who seeks for methods without having a definite problem in mind seeks for the most part in vain". Yet, that is what precisely the chapter does!

Is it possible to write a different screen play for this script? Let me try.

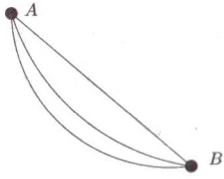
The teacher shows a video in which a group of children riding on a roller coaster. She asks: Roller coaster rides are thrilling. What makes them thrilling? Is it the high speed or the high acceleration? Clearly, it is not the high speed – if it were,

airplane rides must be more thrilling since they travel at speeds much higher than the roller coaster and we hardly feel the speed in an airplane. Hence the thrill element is the acceleration – rate of change of velocity. The quicker you attain the high speed, the higher the acceleration. A roller coaster with maximum thrill element must be designed in such a way that it reaches the top speed in the shortest possible time. Roller coasters do not have motors to propel them – they move under gravity. In the initial drop, the roller coaster reaches a bottom point from a height and the speed it attains at the bottom point depends on the height and also the time it takes to reach the bottom. So can we determine the path it should travel so that it reaches the bottom point in the shortest possible time? We can have several different paths from the top point to the bottom point. Among the paths given below, can you guess in which path will the ball released at the point A will reach B in the shortest possible time? We see that the inclined



plane is the slowest! Determining the path of quickest descent – Brachistochrone – was a problem raised by Johann Bernoulli in the 17th century. Given two points A and B in a vertical

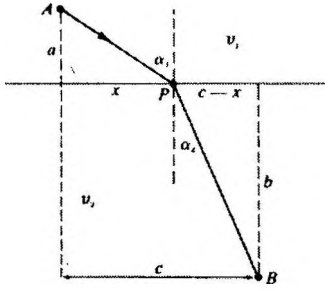
plane, what is the curve traced out by a point acted on only by gravity, which starts at A and reaches B in the shortest possible time?



Johann Bernoulli posed the problem as a challenge to fellow mathematicians. The solution was found by four of the leading mathematicians of the time, Leibniz, L'Hospital, Jakob Bernoulli (brother of Johann Bernoulli) and Newton. On 29 January 1697 the challenge was received by Isaac Newton, who found it in his mail, in a letter from Johann Bernoulli, when he arrived home from the Royal Mint at 4 p.m., and stayed up all night to solve it and mailed the solution anonymously by the next post. Upon reading the solution, Bernoulli immediately recognized its author, exclaiming that he recognizes a lion from his claw mark. Johann Bernoulli had solved it himself and was proud of his solution. He used a very clever approach.

Fermat's principle says that light takes the shortest time to travel between two points. It is also known that the speed of light varies with the density of the medium. Suppose that we have two mediums in which the speeds of light are v_1 and v_2,

what path would light take in traveling from a point A in one medium to a point B in the other medium? From the figure,



we see that we need to minimize

$$T = \frac{\sqrt{a^2 + x^2}}{v_1} + \frac{\sqrt{b^2 + (c - x)^2}}{v_2}$$

We have seen that we can determine this using methods we learned in the Chapter on Maxima and Minima. We differentiate T with respect to x and equate the derivative to zero:

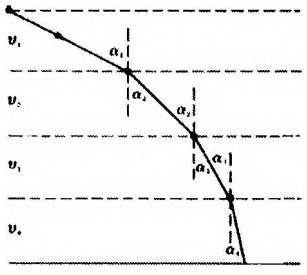
$$\frac{dT}{dx} = \frac{x}{v_1\sqrt{a^2 + x^2}} - \frac{c - x}{v_2\sqrt{b^2 + (c - x)^2}}$$

Equating this to zero, we get

$$\frac{x}{v_1\sqrt{a^2 + x^2}} = \frac{c - x}{v_2\sqrt{b^2 + (c - x)^2}}$$

$$\frac{\sin\alpha_1}{v_1} = \frac{\sin\alpha_2}{v_2}$$

Thus, if we have a stack of mediums, light traveling through

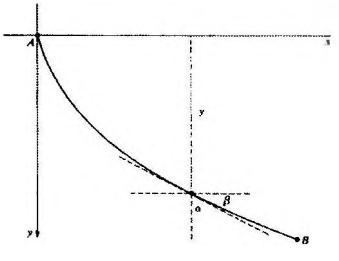


them satisfies the condition that

$$\frac{\sin \alpha_1}{v_1} = \frac{\sin \alpha_2}{v_2} = \cdots = \frac{\sin \alpha_n}{v_n}$$

In other words, $\dfrac{\sin \alpha}{v}$ is a constant. You may recall from Physics, this is called Snell's law of refraction.

Thus if an object travels satisfying the above condition, it must be traveling in the shortest possible time. Now in the quickest



path between A and B, when the object is traveling only under gravity, the loss of potential energy is equal to gain in

kinetic energy. Thus, we have

$$\frac{1}{2}mv^2 = mgy$$
$$v = \sqrt{2gy}$$

But $\sin\alpha$ can be expressed in terms of the derivative as

$$\sin\alpha = \frac{1}{\sqrt{1+y'^2}}$$

Thus,

$$\sqrt{2gy(1+y'^2)} = \sqrt{2gc}$$
$$\frac{dy}{dx} = \sqrt{\frac{c-y}{y}}$$

This is a *Differential Equation.* If we can find a function $y = f(x)$ that satisfies the above equation, we would get the equation of the quickest path.

We write the above as

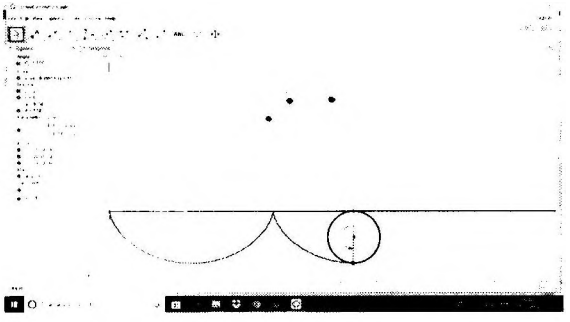
$$dx = \sqrt{\frac{y}{c-y}}dy$$

We already know how to solve this. Using the substitution $y = c\sin^2\theta$ and observing that $y = 0$ when $x = 0$, we obtain the solution

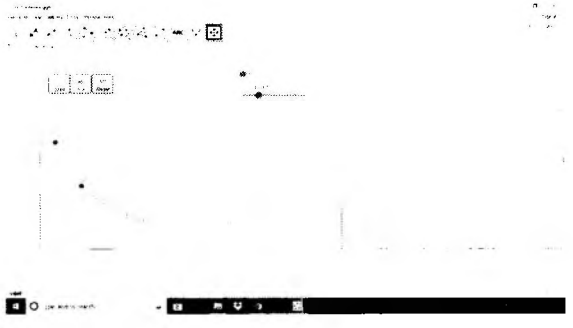
$$x = \frac{c}{2}(2\theta - \sin 2\theta)$$

$$y = \frac{c}{2}(1 - \cos 2\theta)$$

This curve is called a *Cycloid.* From the name of the curve, we can guess that the curve may be some how related to a circle.



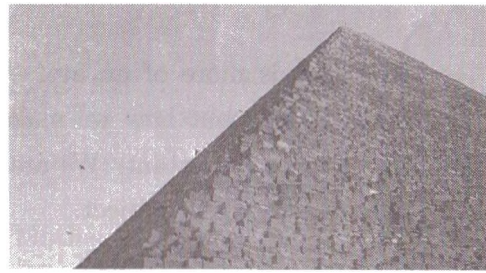
Indeed it is.



It is the curve traced by a point on the circumference of a circle as the circle rolls without slipping on a straight line: This curve also has another amazing property. Irrespective of at which position you start on the curve, you reach the bottom point in the same time.

Not only the opening of a movie important, so is its closing - especially if you are planning a sequel. To keep us intrigued, Bahubali I ended with a tantalizing, shock inducing scene and made us wonder: "Why did Kattappa kill Bahubali?" Similarly, we need to keep the students enticed and looking forward to the next class. How can I do that? Like this:

"There is a close connection between the gold chain that hangs around your neck and the construction of pyramids. We will see that in our next class!"

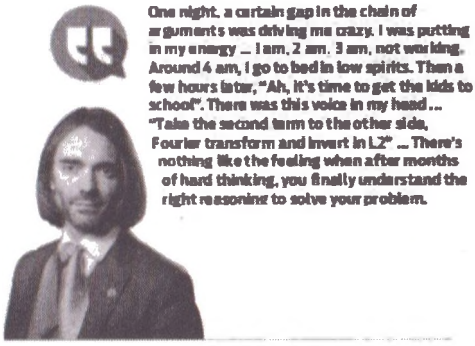


I hope you liked the Screen play!

In essence, we need the following for an effective screen play:

- Why before What
- Ask the right QUESTIONS than giving right answers
- Introduce some history
- Facilitate learning through exploration
- Use technology to visualize and experiment
- Lead the students to experience *Aha* moments

What is an *Aha* moment? Let us hear from Cedric Villani, a French mathematician and a Fields medalist.


One night, a certain gap in the chain of arguments was driving me crazy. I was putting in my energy ... I am, 2 am, 3 am, not working. Around 4 am, I go to bed in low spirits. Then a few hours later, "Ah, it's time to get the kids to school". There was this voice in my head... "Take the second term to the other side, Fourier transform and invert in L2" ... There's nothing like the feeling when after months of hard thinking, you finally understand the right reasoning to solve your problem.

Teaching is more of an art. What we say in the classroom is important but how we make students think is a thousand times more important. We are actually sales persons – selling the beauty of our subject.

George Polya is a Hungarian mathematician whose book *How to Solve It* revolutionized mathematics teaching and learning. His advice to teachers:

- Be interested in your subject
- Know your subject
- Know about learning: The best way to learn anything is to discover it by yourself
- Put yourself in your students' place to see whether your style is effective
- Encourage "guessing"

- Encourage pattern discovery
- Learning is an iterative process – making mistakes is OK
- Let your teaching be an "engaging movie"
- Finally, teach students the history of the subject how and why it was discovered – Most of the time, it was discovered to solve *a* PROBLEM.

You should definitely read his books *How to Solve It* and *Mathematics and Plausible Reasoning.* Another author you should read is Jearl Walker – his book *The Flying Circus of Physics* is about curious events and effects of the everyday world.

How to make teaching come alive? See the videos of Physics classes by Professor Walter Lewin on YouTube.

When you expose the beauty of Mathematics, and make every one of your class an engaging movie, your students would definitely fall in love wouldn't they? After 25 years, many of them would be telling their kids "I had this wonderful Mathematics teacher who brought the subject to life and made me fall in love with Math!"

Evolution of Equations [2]

R. Sivaraman
(National Awardee for Popularizing Mathematics among masses)
Associate Professor of Mathematics, D G Vaishnav College, Chennai
Founder, Pic Mathematics Association
Email: rsivaraman1729@yahoo.co.in

Ever since, humans began to find ways for better living by using natures resources and laws, the idea of Equations should have originated in one way or another. Apparently, no one can possibly predict when and how the idea of equations had evolved. But we can be sure enough that like other important ancient discoveries which had been immensely useful for human progress, the idea of equations and its solving procedures had been more beneficial than ever and will continue to fascinate scientists till this planet survive.

Every equation has a deep idea behind it. It carries beautiful stories and convey the message of the effect that we desire. So equations are so important for knowing lost information and for deciding there-after. It is widely accepted that Theory of Numbers and Geometry were the first two disciplines that originated in mathematics. Once, these two were engaged effectively, the concept of equations emerged simultaneously because equations conveyed the answer for our investigations related to any scientific phenomena. The history

[2]Prof. R.C.Gupta Memorial Lecture at 52nd Annual Conference Association of Mathematics Teachers of India (AMTI)

of mathematical equations did start, however, solely with the benefit of humankind in mind. The first equation solvers attempted nothing more than to address specific everyday needs.

Around fourth millennium BC, the Sumerian civilization came in to existence at Mesopotamia, the land between the rivers Tigris and Euphrates. Nearly half a million cuneiform tablets and other archaeological artefacts found in this area inform us about the story of a society with organized agriculture, impressive architecture, vibrant political and cultural history. Then, as today, this fertile land was prone to invasion from many directions, resulting in a frequent change of ruling populations. It was through some of these tablets we have the first evidence of using equations, probably for measuring lengths for constructing houses and buildings.

Roughly between 2000 and 600 BC, considering Babylon city as their capital, a sect of people flourished. They are referred in history as Babylonians. The rapidly evolving Babylonian society required massive records of supplies and distribution of goods. Computational tools were also needed for business transactions, for agricultural projects involving the partitioning of lots, and for the making of wills. To this end, the Babylonians developed the most sophisticated mathematics of that time. The texts of scores of cuneiform tablets demonstrate that the Babylonians not only mastered a variety of arithmetic manipulations, but literally anticipated more advanced algebra.

Having said this, Babylonians did not truly use the concept of algebraic equations in the same way we do today. Rather, they stated problems and solved them rhetorically, in the language of

ordinary discourse. In other words, one problem after another was solved by precise verbal instructions, but no pattern or formula was ever identified as a general procedure. There is little doubt that these mathematical problems first appeared in the context of the society's need to allocate up lots of land. The words used for the unknown quantities one needed to solve for were: us (length), sag (width), and asa (area), even when no mensuration was involved.

The simplest equations one can formulate are the ones called linear (they are represented by straight lines when graphed). In modern notation these are equations of the type $2x+5=9$, where x represents the unknown. To solve an equation means to find a value of x for which the equation holds true (in the above example, the solution is $x = 2$, since $2 \times 2 + 5 = 9$). Several tablets contain problems whose solutions are applied to society in one way or other.

Sometimes to find the answer, one needed to solve for the value of two unknowns. For instance, in one problem the values of the width and length are called for if one-quarter of the width plus the length are equal to 7 hands (a unit of length), and the length plus the width are equal to 10 hands. Using the present day algebra, if we denote the length by x and the width by y, this problem translates into the system of two linear equations: $\frac{y}{4} + x = 7$ and $x + y = 10$. The Babylonian scribe notes correctly that a length of 6 hands, $x = 6$ (or 30 fingers, one hand being equal to 5 fingers) and a width of 4 hands, $y = 4$ (20 fingers) satisfy both equations.

Linear equations featured even more prominently in the mathematics of ancient Egypt. Apparently the Babylonians

found them too elementary to deserve detailed documentation. Much of our knowledge of Egyptian mathematics comes from the fascinating Ahmes Papyrus. This large papyrus (about eighteen feet long) currently resides in the British Museum (except for a few fragments, discovered unexpectedly in a collection of medical papers, that are in the Brooklyn Museum). The papyrus was purchased by Scottish Egyptologist Alexander Henry Rhind in 1858 and is consequently often referred to as the Rhind Papyrus (figure 1).

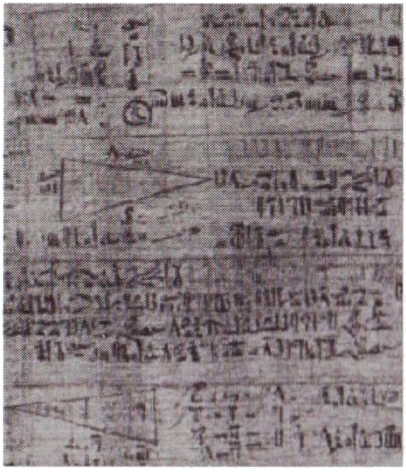


Figure 1

According to the scribe Ahmes's own testimony, he copied the papyrus around 1650 BC from an original document that had been written a couple of hundred years earlier (during the rule of King Ammenemes III of the Twelfth Dynasty). The papyrus, described by British scientist D'Arcy Thompson as "one of the ancient monuments of learning," contains eighty-seven problems. These are preceded by a table of "recipes" for divisions and an introduction. The introduction describes

the document somewhat grandiloquently as "The entrance into the knowledge of all existing things and all obscure secrets." The problems that Ahmes presents and solves, on the other hand, deal mostly with a variety of practical issues, from the fair partition of loaves of bread to the slope of pyramids. The unknown is called aha, meaning "heap." For example, problem 26 calls for the value of aha if aha and its quarter are added to become 15. In modern notation we would formulate the equation $x + \frac{x}{4} = 15$, to which the answer is, as Ahmes correctly finds, $x = 12$.

There are also four lesser documents preserving Egyptian arithmetic: the Moscow Papyrus and the Berlin Papyrus (named for the places they are kept), the Kahun Papyrus (named for where it was found), and the Leather Roll (named for its composition). The Moscow papyrus is sometimes called the Golenischev papyrus after the Russian V. S. Golenischev, who purchased it in 1893 from two Egyptian brothers who found the tomb at Deir el-Bahri. It measures 8 centimeters high and 540 centimeters wide and contains 25 problems and their solutions. The most unusual are the tenth, which seems to give the area of the surface of a hemisphere, or perhaps a cylinder, and the fourteenth, which gives the formula for the volume of the frustum of a pyramid.

The knowledge of how to solve linear equations was not exclusive to the Middle East. The impressive Chinese collection Nine Chapters on Mathematical Art (Jiu zhang suan shu) was composed sometime between 206 BC and AD 221 and was based on a yet earlier collection. In chapter 8 of Nine Chapters, we find problems that involve no fewer than three linear

equations with three unknowns, all solved brilliantly. Several Chinese mathematicians like Zhang Qiujian, Wang Xiaotong, Jia Xian, Li Zhi, Qin Jiushao, Guo Shoujing, Yang Hui, Zhu Shijie, Cheng Dawei have provided remarkable insights in solving equations of all sorts whose solutions were used often for practical purposes as Babylonians or Egyptians.

The next level up, in terms of the intricacy of algebraic equations, is represented by quadratic equations. The extra complication is introduced by the fact that in such equations the unknown, x, appears squared, as in $3x^2 + x = 4$. To the novice this may not look like a dramatic change, yet quadratic equations are actually more difficult to solve than linear ones.

It is often claimed that the Babylonians (about 400 BC) were the first to solve quadratic equations. This is an exaggeration, since the Babylonians had no notion of 'equation'. What they did develop was an algorithmic approach to solving problems which, in our terminology, would give rise to a quadratic equation. The method is essentially one of completing the square. As mentioned earlier, since ancient civilizations primarily considered using mathematics for practical purposes, all problems posed in Babylonian (or Egyptian) had answers which are positive values. They never considered negative numbers as we do today, since the solutions to their problem often represent the length or height of an object. In about 300 BC Euclid developed a geometrical approach which, although later mathematicians used it to solve quadratic equations, amounted to finding a length which in our notation was the root of a quadratic equation. Euclid had no notion of equation or coefficients but he worked with purely geometrical quantities as

is the case throughout the thirteen volumes of his monumental work The Elements.

The Great School of Alexandria produced many outstanding mathematicians over seven centuries. One of the most original thinker of Alexandrian School was Diophantus who is sometimes referred as Father of Algebra. Though not much of his life is known, we come to know that he lived around 250 AD. We come to know of his ingenious ideas through his treatise titled Arithmetica which like Euclids Elements had thirteen volumes. Unfortunately, only six of those volumes had survived due to captivation of Alexandrian Library in the seventh century. An Arabic translation of what may be four more books (attributed to the ninth-century mathematician Qusta Ibn Luqa) was miraculously discovered in 1969.

In spite of receiving the honourable title Father of Algebra, Diophantus wrote mainly on theory of numbers as is evident from his remarkable treatise Arithmetica. Nevertheless, the contributions of Diophantus certainly represent a crucial stage in the evolution of algebra that is intermediate between the purely rhetorical style of the Babylonians and the symbolic forms of equations (e.g., $4x^2+x = 5$) we use today. Diophantus demonstrated incredible creativity and expertise in solving many problems in Arithmetica. Yet he considered only positive numbers as solutions to the equations. Even among positive numbers he only considered what is called rational numbers which are quotient of two integers. Through his work in Arithmetica it is fairly clear that he knew at least half a dozen ways of solving a given quadratic equation.

But today, Diophantus is best remembered for equations

Figure 2

bearing his name: Diophantine Equations. These equations are quite challenging and charmful for a mathematically inclined person, since it counts only the number of whole number solutions to the given equation. Usually in those problems, the number of equations will be less than the number of variables involved, so we may get multiple or even infinite number of solutions. It is this facility, that these equations are studied more extensively by later mathematicians.

As an example, the equation $12x+31y = 636$, has two variables x and y. But there is only one equation for us to solve. The idea is to solve this with the condition that x and y should both be positive integers. It is not very difficult to observe that $x = 22, y = 12$ is the only possible solution for the given equation. Hence, it admits only one solution in positive integers. Mathematicians became interested in counting number of such finite solutions, since, we get infinitely many solutions if we remove the restriction that x and y are positive integers. Such problems are called Diophantine Problems and the mathematical study of such problems is called Diophantine Analysis. Huge amount of research is carried to this day in this branch of mathematics.

But above all these reasons, for converting symbols to algebra, he is normally hailed as Father of Algebra In fact, a sixth century collection known as The Greek Anthology contains six thousand epigrams. One among those provide the information about the scanty record of life of Diophantus, which was believed to be etched as his epitaph. When you go through the details and solve it, you can find how many years did the great Diophantus lived on the planet Earth.

Here lies Diophantus,' the wonder behold. Through art algebraic, the stone tells how old: 'God gave him his boyhood one-sixth of his life, One twelfth more as youth while whiskers grew rife; And then yet one-seventh ere marriage begun; In five years there came a bouncing new son. Alas, the dear child of master and sage After attaining half the measure of his father's life chill fate took him. After consoling his fate by the science of numbers for four years, he ended his life.

Upon assuming x to be age of Diophantus, from the above verse, we find that

$$x = \frac{x}{6} + \frac{x}{12} + \frac{x}{7} + 5 + \frac{x}{2} + 4$$

giving $x = 84$. Thus, if this epigram is believed to be true, then we can conclude that Diophantus lived for 84 years. What an ingenious way of remembering the father of algebra?

After Greek mathematicians, Indians took a big stride in solving various forms of equations. Ever since the mathematical interpretation of place value concept using the number zero was introduced by Aryabhata I during fifth century AD, the mathematics went to a new level of sophistication. Mathematicians like Brahmagupta (598-665

AD) worked extensively on solving various types of equations, the most notable among them being so called (misnamed) Pells Equation, in which he gave a recursive formula to generate infinitely many solutions. There is a legend stating that according to Brahmagupta A person who can, within a year, solve $x^2 - 92y^2 = 1$ is a mathematician.

However, Brahmagupta was mainly credited for admitting negative numbers as solutions to the equations that arose in physical problems. He called negative solutions as debts and positive solutions as fortunes. Another important contribution of Brahmagupta is to have provided the arithmetical laws for the number zero. This allowed him to frame addition and multiplication rules like Negative times Negative is Positive. These concepts as it turned out, played a vital role in understanding the nature of solutions of a given equation. Apart from Aryabhata I and Brahmagupta, various Indian mathematicians like Bhaskara I, Mahavira, Prthudakasvami, Sankara Narayana, Sridhara, Aryabhatta II, Sripati, Bhaskara II, Madhava, Paramesvara, Nilakantha wrote about solving equations in spectacular ways. The Arabs did not know about the advances of the Indian mathematicians so they had neither negative quantities nor abbreviations for their unknowns. However, mathematicians like Al-Khwarizmi, Omar Khayyam the famous poet and mathematician solved equations through geometrical ways.

The man who literally gave algebra its name was Muhammad ibn Musa al-Khwarizmi (ca. 780-850). The book he composed in BaghdadKitab al-jabr waal-muqabalah (The Condensed Book on Restoration and Balancing) became synonymous with

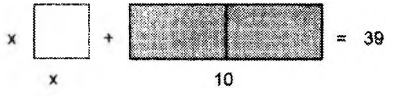
the theory of equations for centuries. From one of the words in the title of this book (aljabr) comes the word "algebra." Even the word "algorithm," used today to describe any special method for solving a problem by following a succession of procedural steps, comes from a distortion of Al-Khwarizmi's name.

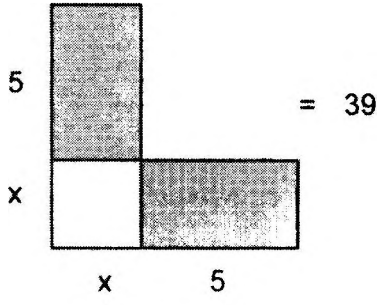
Figure 3: Al-Khwaizimi, widely regarded as the father of Algebra

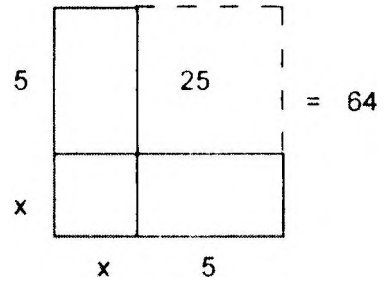
While Al-Khwarizmi's book was not particularly groundbreaking in terms of its contents, it was the first to expose in a systematic way the solutions of quadratic equations. The word aljabr, meaning "restoration" or "completion," referred to moving negative terms from one side of the equation to the other, as in transforming $x^2 = 40x - 4x^2$ (by adding $4x^2$ to both sides) in to $5x^2 = 40x$. So, great was the influence of Al-Khwarizmi's book that he was called as Father of Algebra.

Al-Khwarizmi gave a classification of different types of quadratics (with only numerical examples of each). The different types arise since he had no zero or negatives. He has six chapters each devoted to a different type of equation, the

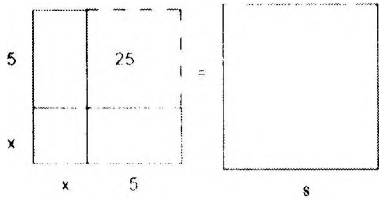
equations being made up of three types of quantities namely: roots, squares of roots and numbers i.e. x, x^2 and numbers. He provided the rule for solving each type of equation, essentially the familiar quadratic formula given for a numerical example in each case, and then a proof for each example which is a geometrical completing the square. As an example, we solve one quadratic equation in the spirit of Al-Khwarizmi. For this, we consider, the equation $x^2 + 10x = 39$.







Note that, by converting the terms of the given equation $x^2 + 10x = 39$ from rectangle to a square in sequence of 4 steps, Al Khwarizmi could solve it by completion of squares



method to arrive at the linear equation $x+5 = 8$ from which he obtained the correct (positive) solution $x = 3$. In this example, the third step refers to Restoration process and fourth step Balancing or Completion process as described to solve the given quadratic equation. We can also note that in this approach, the other solution $x = -10$ is neglected since the variable x is considered as side of a square.

Another Arab mathematician Abraham bar Hiyya Ha-Nasi, often known by the Latin name Savasorda, is famed for his book Liber embadorum published in 1145 which is the first book published in Europe to give the complete solution of the quadratic equation. Any high school student of current education system could easily write out the formula for finding the roots of a general quadratic equation of the form $ax^2 + bx + c = 0$ where $a \neq 0$. Indeed, such a formula exists and it is given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Indian mathematician Sridhara had already provided the same expression for extracting roots of a quadratic equation in 9th Century AD, but his formula unfortunately was not known neither by Arabs nor by Europeans during this time.

For a period of more than three thousand years beginning from early Babylonians to custodians of Science, The Arabs, finally humanity knew how to solve a general quadratic equation in

terms of its co-efficients by using four arithmetical operations and extraction of roots. These were the only possible operations that one can make while trying to solve an equation of given degree. The process of determining the roots of a given equation by four standard arithmetical operations and extraction of roots by using its co-efficients is referred to as Solving by Radicals.

Having completely solved the quadratic equations, the focus of mathematicians now turned to seek the methods of solving a cubic equation of the form $ax^3 + bx^2 + cx + d = 0$ in terms of solving by radicals. The third degree polynomial equations are referred as cubics and those of fourth degree as quartics. The fifth degree polynomial is referred as quintic.

A new phase of mathematics began in Italy around 1500. In 1494 the first edition of Summa de arithmetica, geometrica, proportioni et proportionalita, now known as the Summa, appeared. It was written by the famous painter Luca Pacioli, although it is quite hard to find the author's name on the book, Fra Luca appearing in small print but not on the title page. In many ways the book is more a summary of knowledge at the time and makes no major advances. Though Pacioli does not discuss neither cubic nor quartic equations, his book became notable for the bold statement For the cubic and quartic equations, it has not been possible until now to form general rules. This outrageous claim made other mathematicians of the period to work vigorously to solve cubics and quartics.

Scipione dal Ferro (1465 1526) an Italian mathematician who held the Chair of Arithmetic and Geometry at the University of Bologna was usually credited with solving special case of a cubic

equation of the form $x^3 + px = q$. It is interesting to note that any general cubic equation of the form $ay^3 - by^2 + cy - d = 0$ can be transformed to the equation $x^3 + px = q$ by the appropriate substitution $y = x + \frac{b}{3}$, where $p = c - \frac{b^2}{3}$ and $q = d - \frac{bc}{3} + \frac{2b^3}{27}$. It is interesting to note that without the knowledge of the negative solutions which the Indian mathematicians had developed, Ferro could not have accomplished this task.

Remarkably, Ferro solved this cubic equation around 1515 but kept his work a complete secret until just before his death, in 1526, when he revealed his method to his student Antonio Fior. Fior was a mediocre mathematician and far less good at keeping secrets than Ferro. Soon rumours started to circulate in Bologna that the cubic equation had been solved. Nicolo of Brescia, known as Tartaglia meaning 'the stammerer', prompted by the rumours managed to solve equations of the form $x^3 + mx^2 = n$ and made no secret of his discovery.

Fior challenged Tartaglia to a public contest: the rules being that each gave the other 30 problems with 40 or 50 days in which to solve them, the winner being the one to solve most but a small prize was also offered for each problem. Tartaglia solved all Fior's problems in the space of 2 hours, for all the problems Fior had set were of the form $x^3 + mx = n$ as he believed Tartaglia would be unable to solve this type. However only eight days before the problems were to be collected, Tartaglia had found the general method for all types of cubics. The formulae Tartaglia discovered were so complicated that he found it hard to remember his own rules for the three cases $ax^3 + bx = c$, $x^3 = ax + b$, $x^3 + ax^2 = b$. To help himself memorize them, he composed some verses to remember the

methods of finding the solution.

News of Tartaglia's victory reached Girolamo Cardan in Milan where he was preparing to publish Practica Arithmeticae (1539) Cardan invited Tartaglia to visit him and, after much persuasion, made him divulge the secret of his solution of the cubic equation. This Tartaglia did, having made Cardan promise to keep it secret until Tartaglia had published it himself. Cardan providing the proof of Tartaglias claim and crediting his discovery, decided to announce the world about the secret solution of the cubic. In 1545 he published Artis magnae sive de regulis algebraicis liber unus (The Great Art or the Rules of Algebra Book One), commonly known as Ars magna the first Latin treatise on algebra. This work is considered by many mathematicians as marking the beginning of modern algebra. Its impact was felt throughout the Europe. The word Ars magna became synonym with the Great Art or best work.

Adopting modern notation, here is Cardan's solution of $x^3 + mx = n$. First, notice that $(a-b)^3 + 3ab(a-b) = a^3 - b^3$. So if a and b satisfy $3ab = m$ and $a^3 - b^3 = n$ then $x = a - b$ is a solution of $x^3 + mx = n$. But now $b = m/3a$ so $a^3 - m^3/27a^3 = n$, (i.e.) $a^6 - na^3 - m^3/27 = 0$. This is a quadratic equation in a^3, so we can solve for a^3 using the usual formula for a quadratic. Now a is found by taking cube roots and b can be found in a similar way (or using $b = m/3a$). Then $x = a - b$ is the solution to the cubic.

After Tartaglia had shown Cardan how to solve cubics, Cardan encouraged his own student, Lodovico Ferrari, to examine quartic equations. Ferrari managed to solve the quartic with

perhaps the most elegant of all the methods that were found to solve this type of problem. Cardan published all 20 cases of quartic equations in Ars Magna.

Finally, with the efforts of Ferro, Tartaglia, Cardan and Ferrari the cubic and quartic equations were solved completely. As in the case of quadratic equations, the solution for the general cubic equation where is given by the following exotic formula:

$$x = \sqrt[3]{\left(\frac{-b^3}{27a^3}+\frac{bc}{6a^2}-\frac{d}{2a}\right)+\sqrt{\left(\frac{-b^3}{27a^3}+\frac{bc}{6a^2}-\frac{d}{2a}\right)^2+\left(\frac{c}{3a}-\frac{b^2}{9a^2}\right)^3}}$$

$$+\sqrt[3]{\left(\frac{-b^3}{27a^3}+\frac{bc}{6a^2}-\frac{d}{2a}\right)-\sqrt{\left(\frac{-b^3}{27a^3}+\frac{bc}{6a^2}-\frac{d}{2a}\right)^2+\left(\frac{c}{3a}-\frac{b^2}{9a^2}\right)^3}}-\frac{b}{3a}.$$

Figure 4

The irreducible case of the cubic, namely the case where, Cardan 's formula leads to the square root of negative numbers, was studied in detail by Rafael Bombelli in 1572 in his work Algebra. In the years after, Cardan's Ars Magna many mathematicians contributed to the solution of cubic and quartic equations.

After the brilliant dal Ferro-Tartaglia-Cardano-Ferrari work, it was only natural to believe that the quintic equation, of the form $ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$, could also be solved by a formula. In fact, with the confidence gained from the Ars magna, the expectation was that the solution was right around the corner, and it prompted some of the sharpest mathematical minds to hunt for this treasure.

The story of the search for a formulaic solution to the quintic

equation in the 250 years following Cardano is one of great episode in history of Algebra. The general solution to a quintic equation defied great minds like Rafael Bombelli, Viete, Gregory, Leibniz, Tschirnhaus, Bezout, Euler, Bring, Vandermonde, Waring, Lagrange, Gauss, Ruffini and several others.

We have to pause here for a moment to appreciate the magnitude of what we had observed so far. The formula for the solutions of the quadratic equation had been known essentially since Babylonian times. The formula for the solutions of the cubic was discovered by dal Ferro, Tartaglia, and Cardano. Ferrari came up with the solutions to the quartic. All of these formulae were expressed by simple arithmetic operations and the taking of roots. Then came two and a half centuries of failed expectations, during which some of the most brilliant mathematicians tried in vain to find such a formula for the quintic. Will quintic be solved by anyone? was the most anticipated question in the mathematical world. Then entered two most important contemporary mathematicians Niels Henrik Abel of Norway and French mathematician Evariste Galois.

While, Abel and Galois had a very similar life style full of sufferings, they both died young, Abel at 27 and Galois at 21. Ironically, both didnt live to see the importance and impact of their work in the growth of mathematics. Nevertheless, they did enough to fix their names permanently in the annals of theory of equations.

It was Abel who first proved that a general quintic could not be solved by four arithmetic operations and extraction of roots

(solvable by radicals). This doesnt mean that all fifth degree equations are unsolvable by radicals. For example, the quintic $6x^5-29x^4+27x^3+27x^2-29x+6=0$ is certainly solvable having five real roots. What Abel proved was that not all quintics are solvable by radicals. Unfortunately, this work got recognized only several years after his time.

It was then Galois turn to make a final punch in the story of solving equations by radicals. During the time when Abel was working at Norway, Galois worked independently at France to come up with the most celebrated theorem of Algebra. Indeed, he proved that any polynomial (equation) of degree greater than or equal to 5, is not solvable by radicals. This settled all questions related to whether a given equation can be solvable by four arithmetic operations and extraction of roots. Thus Galois provided the final solution which had defied greatest minds for several centuries. Abel and Galois both died young leaving

Figure 5: Abel

behind great mathematical treasures.

For establishing this fact, Galois introduced a new branch of mathematics called Group Theory which embodies the study of symmetries. Now the branch created by Galois is widely

Figure 6: Galois

applied to almost all branches of Science and Technology.

Abel and Galois proved that the general quintic couldn't be solved by a formula that involves simple operations on the coefficients, not that it could not be solved at all. Yet the actual method of solution remained elusive. Mathematicians Kronecker and Hermite around mid-nineteenth century proved that quintics can be solvable using elliptic functions, but they could not know how to connect elliptic functions with the Theory of Groups that Galois had described. Then came Felix Klein who made ground-breaking contribution which settled the problems of solving equations completely.

In 1884, Klein published a massive tract with the outlandish title Lectures on the Icosahedron and the Solution of Equations of Fifth Degree. How are these two topics related? Icosahedron is a polyhedron with 12 vertices, 20 faces and 30 edges. Klein first showed that there are precisely sixty rotations that leave the Icosahedron unchanged. There are also exactly sixty even permutations among 120 possible permutations with 5 possible solutions to a quintic equation. Klein proved that the

icosahedral group and the permutations group are isomorphic.

But, recall that Galois's proof on the solvability of equations relied entirely on the classification of equations according to their symmetry properties under permutations of the solutions. The unexpected link between permutations and icosahedral rotations allowed Klein to weave a magnificent tapestry in which the quintic equation, rotation groups, and elliptic functions were all interwoven. Just as the completion of a jigsaw puzzle reveals the full picture, the fundamental interconnections discovered by Klein provided the definitive answer as to why the quintic could be solved by elliptic functions.

The unifying power of group theory was so overwhelming that by the end of the nineteenth century it was becoming clear that its reach would overflow the boundaries of pure mathematics. Physicists, in particular, were starting to take notice. First, through Einstein's theory of general relativity, geometry was recognized as a key property of the universe at large. Then symmetry was identified as the foundation from which all the laws of nature ultimately spring. These two simple truths virtually guaranteed that the search for an all-encompassing theory of the cosmos would turn largely into a search for underlying groups.

In modern times, we could solve all sorts of equations through numerical methods with nice algorithms written down. Any super computer could solve equations in flash of a second and can even plot the solutions graphically.

In accordance with Charles Darwins Evolution theory of various

species (inclusive of humans), the Equations also have evolved beautifully over several millennia, in conveying the truth of the Universe, thereby benefitting the mankind. Thus, the Evolution of Equations has a direct influence of Evolution of Human History in general.

I conclude this long journey of theory of equations with the following epigram which I had tried to extend from Galileo.

Mathematics is the Language of Universe – Galileo

If Mathematics is the Language of Universe, then Equations are the Language of Mathematics

What is Mathematics? Six Cultural Fundamentals

Prof. V. Krishnamurthy

Formerly of BITS, Pilani and Former Executive Vice President, AMTI.

Section 1: P-L-E-A-S-E

It is not uncommon to hear people saying they are scared of the very mention of Mathematics. This is mostly due to the fact of their childhood experience of their Mathematics teacher. In younger ages the very process of teaching of Mathematics that we were exposed to could have been the cause of this negative reaction to Mathematics. It is in the teaching of Mathematics that one experiences the need for very good teachers. In most other subjects the effects of wrong teaching or of the teaching of wrong ideas can be countered in later years by the supply of the correct information through more authentic methods of communication. But in the case of Mathematics it is not the wrong facts that have to be erased and corrected but *wrong methods of thinking* that one has been trained to follow as a child or teen-ager. Once a certain line of mathematical thinking or calculation is imprinted in the formative minds of the youngster one would find it difficult, sometimes impossible, to change it to a different way of thinking in later years. Just for example, the child who has learnt the multiplication tables in the form:

8 x 1 is 8, 8 x 2 is 16, 8 x 3 is 24 .....

would find it uncomfortable and near-difficult to think of the multiplication table as

1 x 8 is 8, 2 x 8 is 16, 3 x 8 is 24 ...

Of course this is a very elementary example.

In the processes of Mathematics most of them is a question of mental practice. By mental practice I do not mean rote-learning. Giving the mind a systematic training through the brain to think in a specified direction in order to analyse a problem is what I refer here as mental practice. In the teaching and learning of Mathematics this mental practice gets a special significance. Because this is what gives the learner in Mathematics that peculiar quality of brain called 'analytical capacity' to almost all mathematicians as a class. But those teachers of Mathematics who emphasize this in their classes and make it a point to bring it into focus in all their mathematics teaching are

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Special Lecture in the 52nd Annual Conference of the Association of Mathematics Teachers of India (AMTI)

not the majority. To teach students to solve problems in a mechanical way so they may score well in written examinations is usually the goal of an ordinary Mathematics teacher. Their number is certainly large. The purpose of this article is to focus on six fundamental cultural aspects of the discipline of Mathematics and to emphasize that teachers as well as learners should be able to agree that mathematical communication is not complete until as many of these six as are relevant to the context are emphasized both in teaching and learning Mathematics. What are these six? These six are: **Precision, Logic, Essentialisation, Abstraction, Symbolism** and **Evaluation**. We shall explain these one by one.

PRECISION: Family and friends of Mathematicians usually accuse them of being too sticky with word meanings and verbal expressions even in their worldly life and activities. But that is exactly what Mathematics culture teaches him. Without precision there is no Mathematics. In Mathematics words should have clearly specified meanings, with no possibility of doubt. That is why Mathematicians insist on proper definitions of every technical word and also demand usage of words and language that leave no room for double or multiple meanings.Precision is the oxygen for the life of Mathematics.

LOGIC: Logical method is the only methodology that Mathematics accepts. It cannot accept anything that is illogical.

ESSENTIALISATION: Getting hold of the essentials or the central knot of a problem is the main business of Mathematics the moment it is presented with a problem. Even in ordinary life we usually find that when we are confronted with a problem and a debate about it, very easily the contenders confuse the issue with extraneous arguments that hide the basic issue under question. Mathematics trains its practitioners to throw away what is not essential to the central knot of the problem and concentrate on the strategy to tackle the core issue without being side-tracked into the bylanes that may be legitimate but still do not contribute to the solution of the problem. This essentialisation is one of the main concepts that a teacher particularly should be emphasizing in most of his classes.

ABSTRACTION: This is something which needs some detailed explaining, especially for the layman. Flowers are beautiful, but what is beauty? It is an abstract quality. In this modern period of science and technology, when everything worthwhile in science has to fall back upon mathematical techniques and symbolism, it is true that mathematics is the abstracting science of concrete situations. Not only does it abstract concrete

situations but within itself it keeps on abstracting its own concrete creations to greater and greater levels of abstraction. The ordinary numbers 1,2,3, ... with which we have to deal every moment of our living are themselves abstract concepts. What is the number 2, for example? It is neither a pair of fingers or a couple of apples, nor is it the symbol '2'. It is the abstract concept which is universally common to all pairs of fingers, all couples of apples and so forth. '2' is only a symbol for it. The concept of the number 'two' or for that matter the concept of any number has arisen by the mathematical instinct of our ancestral man 'abstracting' – not perhaps knowing that he is laying the foundations of mathematical abstraction – from the concrete situation of pairs of objects and other multiples of objects seen by him. Secondly abstraction is an every-day process that mathematics and mathematicians are engaged in. Abstraction, generalisation, finding common rules and procedures which are applicable and relevant to many varied situations and contexts are processes that have become unique to the discipline of mathematics. In fact this quality of mathematics which continually abstracts from concrete situations is what distinguishes Mathematics from not only other sciences, but in fact all disciplines.

SYMBOLISM: This can be easily called the' Trademark' slogan of Mathematics. The moment a problem is presented to Mathematics whether it is from within the world of Mathematics or from outside, the first thing that Mathematics does is to understand the problem by symbolising the key factors that constitute the problem. In addition to focussing on the key elements and their mutual relations that are relevant to the problem this ensures that extraneous factors do not cloud the issue on hand.

EVALUATION: Once we have precisely isolated the key factors of the problem and logically grasped the key relations that model the different factors constituting the problem and have caught hold of the essential core then what stares before us is the complex of relations and equations between the different symbols that we have used. Now comes the mathematical evaluation that is likely to unravel the solution to the question posed. This process of evaluation may probably be the longest one in the whole project of solving the problem. It may also need the help of several techniques from various parts of Mathematics. Usually mathematics classes spend all their time on this step of evaluation and consequently spend very little or no time on the other five steps that led to the evaluation state. In actuality many people, – and that includes most of the average teachers – think that Evaluation is all that Mathematics is about. It is this mistaken view that precludes the stu-

dent from understanding the different cultural fundamentals of mathematics And it is the dreary protracted emphasis on evaluation and calculation that drives away students from Mathematics. Mathematics is not only Evaluation but also all the other five!

We shall illustrate all this by taking two simple examples, both from the field of Combinatorics – because this is the area where no technical knowledge of the higher academic levels of mathematics would be demanded, at least in the two problems that we shall take up.

Section 2: PROBLEM ON FRIENDS AND STRANGERS

Since this is a mathematical problem on 'Friends and Strangers', first, in line with the necessity to be precise in all mathematical discussions, we need to agree on a specific definition for 'friend' and 'stranger'. Two persons shall be defined as 'friends' if they have ever met and shaken hands with each other. And two persons shall be defined as'strangers' if they are not friends. These definitions will lead to the situation that in respect of any two persons in the world, there could be no doubt about the fact whether they are friends or not. The fact is irrefutable, even if we are not aware of the fact. Now we shall state the following

PROBLEM: Six persons are seen at a place. We do not know which pairs of them are friends and which pairs of them are strangers. But now Mathematics assures us that out of the six, always, at least three are mutual friends (pair by pair) and if not, at least three are mutual strangers (pair by pair). How is this? For convenience we shall refer to this as 'The Friendship Theorem' – though the name "Theorem' applies to it only after we can prove it, which is what we are going to do in the sequel.

One clarification. Three or more are said to be mutual friends, if *every* pair of them are friends. And in the same way, three or more are said to be mutual strangers if *every* pair of them are strangers. Reader, Note that at every stage the precision requirement of the culture of Mathematics is being taken care of by carefully worded definitions. Now let us start the discussionon of the problem.

First, what is the relationship of this problem with Mathematics? Where does Mathematics come in here? These may be our first reaction. But once we catch hold of the essentials of this problem we will come to know where logic and Mathematics come in here. Notice that no names of the people are given. The names are not relevant. What is relevant is only the fact

that, whatever pair of persons among the six you may take, the question is whether they are friends or strangers. And further, notice the use of the words 'at least three'. It could therefore be three or four or five or six. Recall the precision that must be involved in every statement of Mathematics.

Let us now analyse the problem in the cultural style of Mathematics. It may appear that there is nothing to hold on to in the problem. This is because there is nothing but 'six persons', 'friends' and 'strangers' in the statement of the problem. This is where the symbolism of Mathematics comes into play. No names are given. So let us name the six persons according to our convenience. Let us call them A, B, C, D, E and F. Now we have some straws to hold on to. More, we can even think of these names as names of points marked on a plane. Thus we have symbolised the six persons with the six points on the plane. Let us continue the task of symbolism. How do we symbolise the relation of 'friendship'?. It is easy to guess the next step. We shall symbolise 'friendship' by saying tha a line joining A and B will indicate A and B are friends. In other words if there is a straight line joining two points, that means the two (persons represented by the points) are friends. If there is no line between two points they are strangers. Now there are several possibilities. (Mathematics will tell you there are

$$2^{15}$$

possibilities. But this is not relevant now).

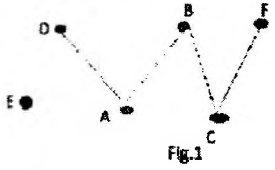


Fig.1

Out of all these possibilities Fig.1 shows one possibility, just to illustrate the agreed symbolism. In this illustration, A and B are friends; A and D are also friends; so also are the pair B, C; and also the pair C,F. Any other pair is only a pair of strangers, because there is no line other than these four. Incidentally, we note also that in this example there is no triad of 'mutual friends'; however there is a triad -B, D and E - of mutual strangers. In fact there are more than one such triad of mutual strangers. There is even a set of four mutual strangers - namely, D, E, B and F. All this bears testimony to the truth of the mathematical assertion that there is one set of at least three mutual friends or one set of at least three mutual strangers. But mark it, *this is no proof* of the theorem. In order to prove the theorem we have to produce a logic that for *every one* of the possibility for the six persons the statement of the theorem is true.

When we talk of 'logic' and 'proof' we should recall one more logical fact of the culture of Mathematics. One example (as was mentioned above) does not prove a theorem, certainly. But one counter-example - wherein we show that the conclusions of a proposed theorem are not valid in this particular

example - is enough to prove that the theorem is not true in general. In other words, one counter-example is enough to disprove a theorem. To disprove the Friendship theorem therefore we need to find a counter-example. But the contention of Mathematics is that no matter what you try you will not be able to find a counter-example to the friendship theorem. We can go on looking at every possible combination of relationships among the six members A, B,C,D,E and F; we will not be able to arrive at a counter example. To look at every such combination of relationships and verify the statement of the theorem for that example would be a stupendous task not to be undertaken. We have to use logic to prove the theorem in general.

The core essentials of this problem are the relationship that defines friends and strangers. Our symbolism tells us this is a matter of deciding whether there is a line joining a specific pair of points among the six points or not. So the question is one of deciding which pair of points among the six has a line joing them and which pair has not. Concentrate on a single point, say, A. How many lines are likely to emanate from A? Five. In any one specific combination of relationships, pictured by a specific diagram, how many of these five lines exist or do not exist? Can we say something general about this, without looking at a specific diagram? We just used the word 'general'. This is where the concept of 'Abstraction' – one of the six fundamentals, we listed as the constituents of the culture of mathematics – comes into play. Again go back to A. From A there could be no line,there could be 1, 2, 3, 4, or 5 lines. Of the possible five lines, AB, AC, AD, AE, AF, which connect the pairs (A,B), (A,C), (A,D), (A,E) and (A,F),the various possibilities are:

no pair is connected and so five pairs remain connected;

one pair is connected and so four pairs remain unconnected;

two pairs are connected and so three pairs remain unconnected,

three pairs are connected and so two pairs remain unconnected;

four pairs are connected and so one pair remains unconnected; and

five pairs are connected and so no pair remains unconnected.

In other words, always, **three or more pairs remain unconnected** or **three or more pairs remain connected**. Looking at this from the point

of view of the number of lines emanating from A, we can say in general that always, out of the possible five lines AB, AC, AD, AE, AF, there are only two distinct possibilities:

CASE 1: Three or more lines exist

CASE 2: Three or more lines do not exist.

The interesting point to be noted here is the fact that the multifarious possibilities have now been reduced to just two cases. This is the power of the mathematical way of abstract thinking. Speaking technically, the principle that has been used here goes by the name of THE PIGEON-HOLE PRINCIPLE. This principle, in this context, says that when five things are divided into two parts at least one part must contain more than half of them. When th five pairs (A,B), (A,C), (A,D), (A,E), (A,F) are divided into two parts by the criterion whether there are (or there are not) lines joining the members of the pair, one of the parts must have at least three in it. This is the application of the pigeonhole principle in this case. *In fact this is the punchline argument* in the solution of the problem of the Friendship Theorem.

We are now ready to go to the final stage of Evaluation. Evaluation does not always mean algebraic calculation or geometric analysis. It is the putting together of all the threads that we have collected, making sense out of it and arriving at conclusions. So let us take up Case 1. Of the five pairs AB, AC, AD, AE, AF we now know at least three exist. Without loss of generality we can take the existing ones to be AB, AC, AD.

A digression on the culture of Mathematics. What does it mean to say 'Without loss of Generality'? Instead of the three lines AB, AC, AD, we have chosen as existing it could be any other three, for instance, AC AE, AF. In that case we can treat them exactly as we are going to treat AB, AC, AD. This is the meaning of saying 'Without loss of Generality'. Whenever in Mathematics we find a situation where there appear several almost identical possibilities which differ only in name and when we are sure that the change of names will not make any substantial change in the argument that follows, we then say 'Without loss of Generality' and choose one of those possibilities to continue the argument.

Since lines AB, AC, AD exist it means A and B are friends, A and C are friends, and A and D are friends. Now look at the triad of points, B, C, and D. Two (and only two) cases arise; namely, Either, one (or more) of the lines BC, CD, BD exist or none ofthe lines BC, BD, CD exist. In the former case,

(again without loss of generality) take BC as existing. Now this BC, with the already existing lines AB and AC tells us that there is a triad A, B, and C who are mutual friends and so the conclusion of the theorem is validated. In the latter case, the triad of points B, C and D do not have any connecting lines and this means B,C and D are mutual strangers. Thus in both cases the statement of the theorem is validated.

We now go to Case 2. In this case, of the five pairs AB, AC, AD, AE, AF we know at least three do not exist. (As before) Without loss of generality we can take the non-existing ones to be AB, AC, AD. The argument is very similar to that of Case 1. Now A and B are strangers; A and C are strangers; A and D are strangers. Look at B,C,D now. Two distinct cases arise. Out of B, C and D at least one pair of them are strangers or, alternatively no pair of them are strangers. In the former case, say (without loss of generality) B and C are strangers. Already A and B are strangers; and A and C are strangers. These three stranger-relationships tell us that A, B and C are mutual strangers. In the latter case since no pair of B, C, D are strangers it means that B,C, D or mutual friends. In either case the statement of the theorem is validated. Thus all cases have been exhausted and the Friendship Theorem stands proved.

Section 2: ABSTRACTION

We said earlier that Mathematics is an abstracting science of concrete situations. We are now going to see how the problem of Friends and Strangers which is a very concrete situation has been abstracted by Mathematicians to a very high abstract level. In this process Mathematics is different from most other sciences. In other sciences you raise questions in order to solve problems arising from concrete situations. But in Mathematics once a concrete problem is on hand, one is not satisfied with just solving that problem but one proceeds further to see how it can be generalised (=abstracted) to a level where it may (or may not !) be applicable to further unknown situations which probably has not arisen yet. To understand this statement one has to go on with the abstraction we are heading to. Many have attested to this surprising 'unreasonable effectiveness' of Mathematics.

When we symbolised the details of this problem we symbolised persons by geometrical points and the relation of friendship as a line joining two such points. We shall now concoct another symbolism, though not much different from the earlier one. Keep the six points as representing the six persons. Join all possible pairs of points. Since there are 6 points there will now be 15

lines joining different pairs of them. Colour these 15 lines with two colours, say, red and blue. Let the blue lines represent the relation of friendship and let the red lines represent the relation of strangership.

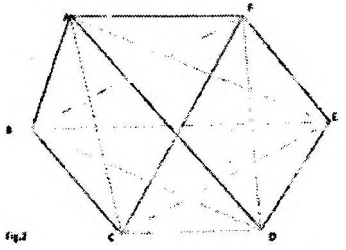


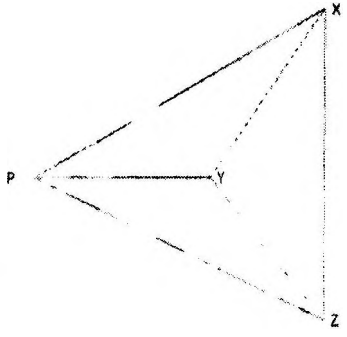
Fig.2 above in which there are 4 blue lines and 11 red lines is nothing but the same diagram as Fig.1 in the new symbolism. Here A,C,E are mutual strangers. Each one of them is red-connected with the other two. In other words, the triangle ACE is a 'red triangle', meaning all the three sides are red. In fact in Fig.1 also A,C, E are mutual strangers; according to the symbolism used in Fig.1, they are totally disconnected and so they are mutual strangers. In our present symbolism the vertices of a blue triangle form a triad of mutual friends and the vertices of a red triangle form a triad of mutual strangers. We can now restate the Friendship Theorem in the following form, called BICOLOUR RAMSEY'S THEOREM:

If all the 15 lines joining the 6 vertices of a hexagon be coloured in any manner with two colours, there will always be at least one triangle, all the three sides of which will have the same colour.

Note that the concepts of 'friends' and 'strangers' have disappeared now. This is the power of mathematical abstraction. The proof of the above theorem is exactly the same as that of the Friendship Theorem. Since that was an introduction to the reader to the six fundamentals of the culture of Mathematics, we spent a leisurely long time to give the proof. Now we shall give the straight-forward unidirectional proof without any digression and one will see the precise way in which mathematical proofs proceed.

Let the colours be red and blue.Focus your attention on the five lines that go forth from a specific vertex, say P. They are coloured red or blue – some

red, some blue. We assert that at least three of these lines must be of the same colour. For, if there are *less than three*, which are red, say, then there are at least three which are blue.



Consult Fig 3 above. Suppose X, Y, Z are the other ends of these three lines, all of the same colour, say red. If either one of XY, YZ, or ZX is red, that line with the two lines from P meeting it at its ends would give a red triangle. If none of XY, YZ, ZX is red, then all three are blue and we have a blue triangle!

The utter simplicity of this logic which so powerfully produces a very interesting conclusion almost from nowhere, is what makes the Friendship Theorem interesting. In 1930, in a paper entitled 'On a Problem in Formal Logic' F.P. Ramsey proved a very general theorem of which the Friendship Theorem is a meek corollary. We shall state Ramsey's original Theorem below, without proof, just to give a simple illustration of what it means to abstract and generalize in Mathematics.

But before we do that, let us question the importance, if any, of the number 6 in both the Friendship Theorem and the Bicolour Ramsey Theorem.

What is the role of the number 6? Suppose we take 7 points and connect the points in all possible ways and colour the lines red and blue. Would a single colour triangle (technically known as *mono-chromatic triangle*) again exist always?. Yes, it would Because six vertices are only part of seven vertices. For six vertices Ramseys theorem says a monochromatic triangle would certainly exist. And this triangle is certainly also a part of the diagram of those seven vertices. And what does this argument really show? For any number greater than six, Bicolour Ramsey theorem would hold.

Now take a number less than six, say five. Five points are given. The connecting lines – they will be 10 in number – are also drawn and they are coloured red and blue. In whatever manner we colour, would a monochromatic triangle always exist? Either we look for a logical proof of this or we look for just one counter-example which will show the non-existence of a monochromatic triangle. Such a counterexample is given in Fig.4, where the five-point diagram has no red or blue triangle.

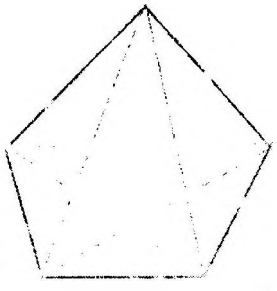


Fig.4

Numbers lower than five will also give the same result. For the same counter-example as Fig.4 with the necessary vertices taken out will suffice as a

counter-example. Incidentally the counterexample for five, given in terms of a graph can be reinterpreted in the language of friends and strangers as follows. In a party of five seated at a round table, each person is a friend of only the two sitting next to him on either side!

The above analysis shows that 6 is the smallest number for which the Friendship Theorem and the Bicolour Ramsey Theorem are true. Therefore 6 is known as a Ramsey number. To be precise this is symbolically written

$$N(3,3;2) = 6$$

. The number '3' on the left hand side indicates that we are talking of monochromatic three-sided figures. The number '3' occuring two times indicates that we are using two colours in the problem. The number '2' indicates that what is being coloured is a two-sided figure, namely a line. The symbol N(3,3;2) stands for the smallest number of points, for which if we colour all their connecting lines (2-sided figures) with two colours, there will always exist a mono-chromatic triangle (three-sided figure).

The general Ramsey Theorem reads as follows: **Let t be a positive integer. Let r, q_1, q_2, ... , q_t be positive integers such that $1 \leqslant r \leqslant q_i$ for every t. Then there exists a smallest positive integer N, (which of course depends on the r and the q's) such that the following holds. Let S be a set with n elements, where $n \geqslant N$. Let the r-subsets (i.e., subsets containing r elements) be partitioned into t distinct classes listed as A_1, A_2, ... A_t. In other words each r-subset of S is precisely in one of these t classes. Then for some $i = 1, 2, ..., t$ there exists a subset X of S with q_i elements such that all r-subsets of X belong to the same A_i.**

Reader, if you are flabbergasted by the obviously terse symbolism of the above theorem that is because you have suddenly been taken up the ladder of abstraction through several rungs. Be consoled you are not alone! But you should at least be curious to know how the above is really an abstraction or generalization however remote, of the Friendship Theorem. It is this way. Take $r = 2$. This means we are interested only in 2-subsets. So if S is the set of vertices of a graph, we are interested in *the edges of the graph*.The use of a general number r instead of 2 is one level of abstraction. And further take $t = 2$. This means that the r-subsets (in this case the edges of the graph) are partitioned only into *two classes*, namely those which are coloured in one colour and those which are coloured in the other colour. The use of t instead of 2 is another level of abstraction. Then Ramsey's theorem says

that *there exists* a smallest positive integer N (in the case of the Friendship Theorem this is 6) which has the following property: If the complete graph on n vertices ($n \geq N$) has its edges partitioned into a red class and a blue class, there exists a triangle (q_i-subset, i.e., a 3-subset; the use of q_i itself is another level of abstraction) all of whose sides belong to one colour.

So the Friendship Theorem is the special case of Ramsey's Theorem, with $r = 2, t = 2, q_1 = 3 = q_2$.

But there is one major difference between the Friendship Theorem and Ramsey's Theorem. We certainly pay a big price for abstraction. While the Friendship Theorem asserts that 6 is the magic number, Ramsey's Theorem only asserts the existence of such a number and it gives no indication of what that number could be or how one calculates or arrives at that number. Calculations of these numbers, called *Ramsey numbers*, is not easy. Mathematicians denote the Ramsey number depending upon r, q_1, q_2, ... , q_t by the symbol

$N(q_1, q_2, ..., q_t; r)$

The Friendship Theorem says $N(3, 3 : 2) = 6$.The general Ramsey numbers for various r's and all possible q's are therefore *known* to exist but the actual determination of them is a continuing research problem. Most of the numbers which are known are for the case $t = 2$. For $t = 3$ the only known number is $N(3, 3, 3; 2) = 17$. Let us understand what this means. On the left hand side symbol there are three 3's. Each 3 indicates a triangle. Since it occurs three times it means we are talking of tricolouring. In short it says the following. If we have a polygon with 17 vertices and 17 edges and if we draw all the connecting lines and if we colour the lines with three colours, in whatever manner we colour them, always, there will exist a mono-chromatic triangle. And in this statement 17 cannot be replaced by a smaller number.

There are many such Ramsey numbers. The research activity on these numbers is continuing. We are told that these have applications in Electronic Engineering and in researches related to DNA.

Note that we started with a simple-looking problem of Friends and Strangers which even appeared as if it has nothing to do with Mathematics but slowly and gradually we have abstracted it to the stage where we find ourselves in the frontier area of the mathematical field called Combinatorics.

Section 3: SECOND PROBLEM

This one, while it also illustrates the play of most of the six cultural fundamentals of Mathematics, will be comparatively easier than the problem

of Friends and Strangers. So we can afford to go a little faster and we shall do so.

PROBLEM: A chess player plays 45 matches in a continuous sequence of 30 days, playing at least one match each day. Though we are not given how many matches he played on each day, prove that he must have played 14 matches in all on some consecutive days.

You will see how important it is to have symbols and symbolism. We do not know how many matches he played each of the days. So let us assume that he played m_i matches on the i-th day, $i = 1, 2, 3, ..., 30$. Obviously,

$$m_1 + m_2 + ... + m_{30} = 45$$

14 matches in all on some consecutive days – is what we have to show. What consecutive days, from what day to what day, – these are unknown. Let us, for a moment, though temporarily, take it that it is probably from, say, the 7th day to, say, the 12th day. This would mean

$$m_7 + m_8 + m_9 + m_{10} + m_{11} + m_{12} = 14$$

Of course this can also be said in a different way without losing the precision. If from the total number of matches he had played from the 1st day to the 12th day, we subtract the total number of matches he had played from the 1st day to the 6th day then that would also be 14. From this observation one thing strikes us. It appears that the total number of matches he had played from the beginning up to a certain day is an important factor in this problem. So let us use a symbol M_i to denote the total number of matches he had played from the 1st day to the i-th day. In other words,

$$M_i = m_1 + m_2 + ... + m_i, i = 1, 2, ..., 30$$

So we have a set of numbers

$$\{M_1, M_2, ...M_{30} = 45\}$$

Call this set A. A has 30 members Now, again go back to the problem and recall: 14 matches in all on some consecutive days – is what we have to show. Consecutive days – from what day to what day? This is not known. So in consonance with mathematical tradition we shall assume that it is from the x-th day to the yth day. Obviously we have assumed $y > x > 1$

Thus it turns out

$$M_y - M_{x-1} = 14$$

So

$$M_y = M_{x-1} + 14$$

Motivated by this, we shall construct another set

$$B = \{M_1 + 14, M_2 + 14, ..., M_{30} + 14 = 59\}$$

This set also has 30 members.

Now comes the punchline of the solution to the problem, namely, the Evaluation aspect. If we try to make a union of the two sets A and B, this should have 60 members. The 30 members in A are *distinct* numbers among the numbers 1 to 45. Why distinct? Because every day the player plays *at least* one match. The 30 members in B are *distinct* numbers among the numbers 1 to 59. Why distinct? For the same reason as above. Thus the 60 members of A and B together are among the numbers from 1 to 59. *The pigeon-hole principle applies now.* So, there has to be *at least one repetition* in the list of members:

$$M_1, M_2, ..., M_{30}; M_1 + 14, M_2 + 14, ..., M_{30} + 14$$

This repetition cannot be in the first 30 members, because they are distinct. Nor can it be among the second 30 members, because they are also distinct. Thus there is an M_y and an $M_x + 14$ such that

$$M_y = M_x + 14$$

This establishes the fact that between some two days, the player has played, in all, 14 matches!

Section 4: REFORMULATION AND ABSTRACTION

REFORMULATION OF THE SECOND PROBLEM: If we pick up any 30 numbers from 1 to 45, there will certainly be two numbers in our pick such that their difference is 14.

(The proof is left to the reader).

ABSTRACTION OF THE ABOVE: *If we pick a set of M numbers from the numbers 1 to N then there will certainly be, in our pick, two numbers with difference 2M-N-1, provided 2M > N + 1.*

MASS PRODUCTION OF SPECIAL RESULTS FROM THE ABSTRACTION:

1. If we pick 55 numbers from 1 to 100 we cannot avoid a difference of 9 between some two numbers.

2. Any selection of 35 numbers from 1 to 50 will certainly have two numbers differing by 19.

.....

Section 5: CONCLUSION

In both the examples of problems we selected for discussion, the fundamentals of the culture of mathematics, namely, Precision, Logic, Essentialisation, Abstraction, Symbolism and Evaluation were in play, sometimes very forcefully. But in both of them the essential core had to do with the pigeon-hole principle. In most of Mathematics, it is this aspect of catching and holding on to the essential core of the problem on hand that gives the research mathematician a whole panorama of excitement and an unveiling of unexplored new finds. We ourselves had this excitement in our meek imitation of exploration of the two problems. At first it appeared as if there is no great mathematics involved. But once we were in, we were led on to more and more exciting avenues of exploration and we carried on. Throughout the mathematical world, over the centuries, hundreds of mathematicians have been carrying on this kind of exploration into uncharted areas and that is what keeps Mathematics alive, useful and invaluable! An ever-increasing abstraction has been the hallmark of Mathematics, particularly in the last two centuries. Many such abstractions were done originally as 'art for art's sake'. The invention of non-Euclidean Geometry or that of Galois fields or, for that matter, Ramsey Theory (there is an abundance of such examples) were all done mainly as art for the sake of art. Many of them turned out to be major tools in the application of mathematics in later years. Such is always the case when mathematics is abstracted by a master. Galois (1811-1832); Jacobi (1804-1851); Cayley (1821-1895); Weierstrass (1815-1897); Cantor 91845-1918); Banach(1892-1945), Polya (1888-1985); Srinivasa Ramanujan (1887-1919), Harish Chandra (1923-1983) are representative names that should remind students of Mathematics and the Sciences that these and several other names and their achievements through the past can make us aware of the rich Mathematics culture of that past that has been handed down to us.

ASSOCIATION ACTIVITIES

1. The regular May workshops were held at four levels
   (a) Primary - 25 registered
   (b) Sub-junior 80 registered
   (c) RMO group 78 registered and
   (d) Teachers 19 registered.

2. Published:
   (a) The Mathematics Teacher of volume 54 1 & 2 released on 18th June 2018
   (b) Junior Mathematician volume 28 (1) released on 10th November 2018 (September issue)

3. Organized and successfully conducted the 50th National Mathematics Talents Examination at five levels Primary (5th & 6th classes), Sub Juniors (7th & 8th classes), Juniors (9th & 10th classes), Inter (11th & 12th classes) and Seniors (Degree classes). Details given in Table 1. In the 50th NMTC this Year there has been a increase of 112 in number of institutions, number of individuals are increase 11148 participants taking test scheduled on 01.09.2018. There was a decrease of 133 individual aspirants this year.

4. We received Rs. 1,75,000/-as grant from HBCSE for RMO, INMO grant from HBCSE for RMO and INMO.
   - RMO exam was held two centers: PSBB KK Nagar and Suguna PIP school, Coimbatore. Original scheduled date was 07.10.2018 but rescheduled to 28.10.2018 due to weather problem (RED ALERT).
   - Revaluation work is in progress - (28 revaluation requests received.)

NUMBER OF INSTITUTIONS REGISTERED: 757

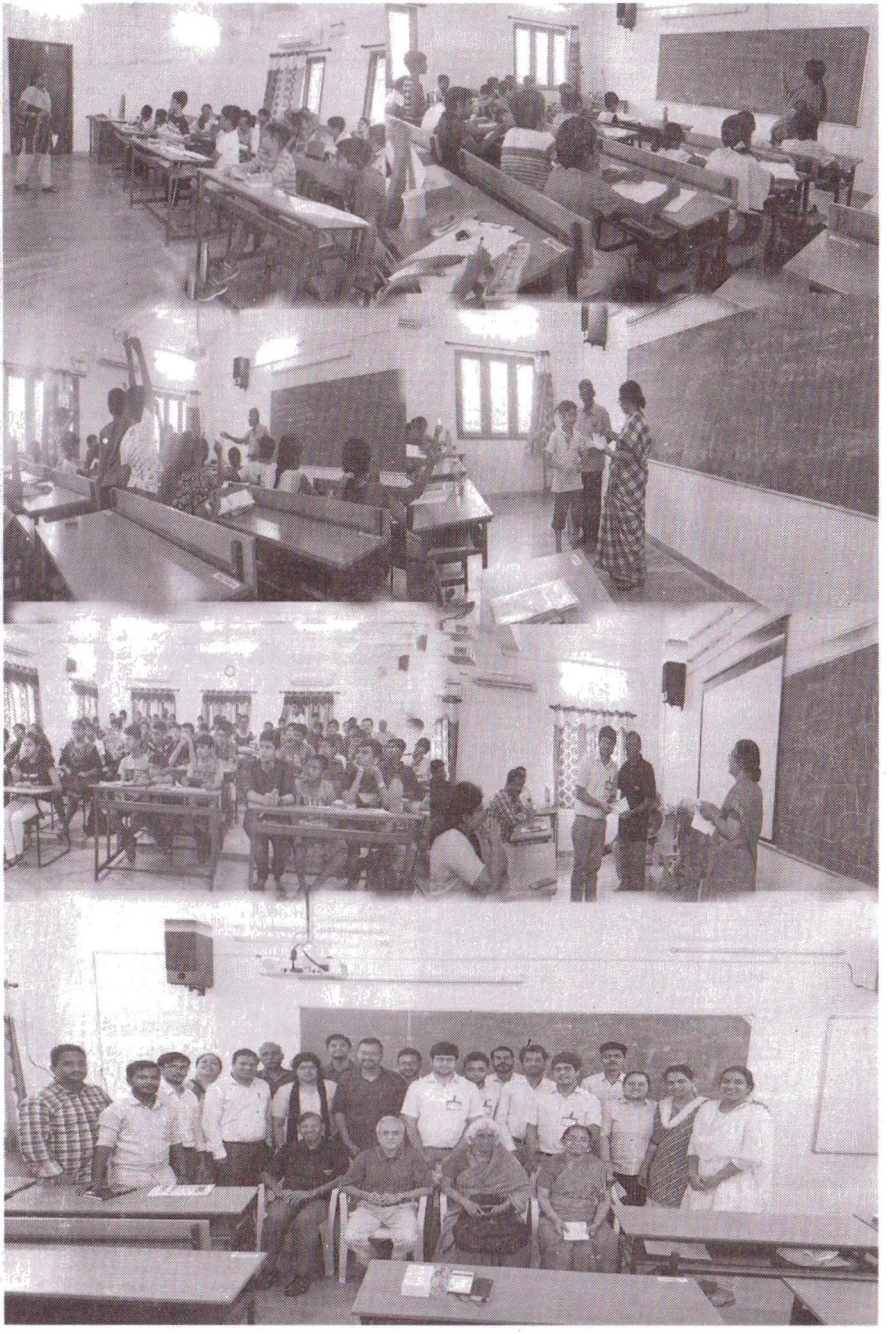
States and the no. of institutions registered:		Languages represented & catered:	LEVELS					
State	**No.**		**Primary**	**Sub-junior**	**Junior**	**Inter**	**Senior**	**Total**
Andhra Pradesh	298	***English***	34576	44189	49398	6419	420	135002
Bihar	1	***Gujarati***	10	12	39	50		111
Chandigarh	3	***Hindi***	45	44	42	44		175
Delhi	7	***Kannada***						
Gujarat	19	***Malayalam***	5	8	16	1		30
Haryana	10	***Maratti***						
Jharkhand	2	***Tamil***						
Karnataka	33	***Telugu***						
Kerala	2	***Total***	34636	44253	49495	6514	420	135318
Madhya Pradesh	8							
Maharashtra	70							
Orisha	10							
Pondicherry	2							
Punjab	2							
Rajasthan	22	***English***	*47*	*124*	*138*	*40*	*3*	*352*
Tamilnadu	109							
Telungana	131	***Total***						
Uttar Pradesh	7							
Uttarakhad	2	***Open quota (States and the no. of institutions registered)***						
West Bengal	19	*A P*	*1*	*Jharkhand*		*1*	*UP*	*30*
		Assam	*1*	*Karanataka*		*34*	*WB*	*11*
Total	*757*	*Bihar*	*11*	*Kerala*		*1*	*M.P*	*7*
		Chandigarh	*8*	*Maharashtra*		*81*		
		Chattisgarh	*1*	*Odisha*		*2*		
		Delhi	*13*	*Punjab*		*12*		
		Goa	*4*	*Rajasthan*		*13*		
		Gujarat	*11*	*Tamilnadu*		*74*		
		Haryana	*30*	*Telungana*		*2*	*Total*	*352*
		Jammu & Kashmir	*3*	*Uttarkhand*		*1*		

Level	Primary	Sub-Junior	Junior	Inter	Senior
Registered	34636	44253	49495	6514	420
Selected for final	3669	4868	3545	710	420

Figure 1: Table 1

MAY 2018 - WORKSHOPS

MAY 2018 - WORKSHOPS



"An important Message"

In tune with the exploding competitive demands all over, we in the association motivate pupils to aspire for excellence in mathematics Education. (A matter of fact routinism that is practised in classes does not realise this goal. Entrepreneurs of various hues exploit this situation. How to use regular concepts to help the students learn in depth analytical thinking and application is the service taken up by the (AMTI for more than 50 years. NMTC 50 is already over and not less than 135670 pupils participated in the 50th held in 2018. To help such aspirants guidance plan has been planned and held more than 14 times by us in various forms. (If late we are holding 4 day workshops for various levels as indicated below. Primary - students of classes 5 and 6, Sub juniors - students of classes 7 and 8, Juniors - classes 9 and 10 (in which even class 11 students can join) and for teachers too. These are orientation classes introducing how to solve problems from known facts at various levels. Class is indicative and not conclusive as talents are not class based!

A student of class 5 opined a few years back after the four day workshop that

"They found these 4 full day Maths. workshops very interesting while the 7 periods a week in the school maths. was very boring!"

This is the objective with which we organize these workshops. Interested people can apply for which notice will be displayed in our **website: amtionline.com** *in January with last date for* **applying to be 1st April.**

Make use of this special service and be benefited.

The annual series of "May workshops-2019" have been planned as per the following schedule:

Students:

Primary Level	-	1st May to 4th May - 2019
Sub-Junior Level	-	8th May to 11th May - 2019
Junior Level	-	15th May to 18th May - 2019
Teachers' Workshop	-	22nd May to 25th May - 2019

Complete details will be available on AMTI website after January 2019.

THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

(Founded 1965; Registered as S.No. 143 of 1965 under the Societies Registration Act XXI of 1860)

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1.	The Charm of Problems Solving	Rs.100/-
2.	Sample Questions and Solutions	Rs. 80/-
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PRESIDENT
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Department of Mathematics
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GENERAL secretary
Dr.R.SANTHI
19,II,link Street, Sadasiva Nagar;
Madipakkam, Chennai 600 099
santhi.educator@gmail.com

TREASURER
Sri V. SUNDARAMURTHY
529/5, Lake View Main Road,
Ayyappa Nagar Extension,
Madippakkam, Chennai
Phone: 9840410545 / 9841070392

EDITOR - Mathematics Teacher
Dr. S. MURALIDHARAN
18, Phase 4, Wood Creek County,
Near Chennai Trade Center
St. Thomas Mount P.O., Chennai-600016
muralidharan.somasundaram@tcs.com

EDITOR - Junior Mathematician
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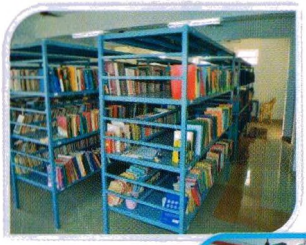

The Association of Mathematics Teachers of India (AMTI)

Regd. Office: B-19, Vijay Avenue, 85/37, Venkatrangam Pillai Street, Triplicane, Chennai-600 005.
Ph : 044-28441523
Project office: Moppedu Road, Alappakkam, New Perungalathur Post, Chennai - 600063
Ph: 044-22792423
e-mail : amti@vsnl.com and support@amtionline.com

To